

أ.1

$$y = \ln(x^2 + 5^x) \Rightarrow \frac{dy}{dx} = \frac{2x + 5^x \ln 5}{x^2 + 5^x}, \quad y = e^x \sin x \Rightarrow \frac{dy}{dx} = e^x \cos x + e^x \sin x$$

ب. الدالة $f(x) = \cos x$ زوجية لأن $f(-x) = \cos(-x) = \cos x = f(x)$

$$\lim_{x \rightarrow 0} \frac{12x - 2 \sin x}{5x} = \lim_{x \rightarrow 0} \left(\frac{12}{5} - \frac{2 \sin x}{5x} \right) = \frac{12}{5} - \frac{2}{5} = 2, \quad \lim_{x \rightarrow 0} \frac{3 \cos 2x}{1-x} = \frac{3 \cos 0}{1-0} = \frac{3}{1} = 3 \quad \text{أ.2}$$

ب. إذا كانت $f(x) = \sec x$ فإن:

$$f'(x) = \sec x \tan x$$

$$f''(x) = \sec x \cdot \sec^2 x + \tan x \sec x \tan x = \sec^3 x + \sec x \tan^2 x$$

$$\text{أ.3} \quad \text{الدالة } f(x) = \frac{2x^2 + 2}{x^2 + 1} \text{ مستمرة على الفترة } (-\infty, \infty)$$

$$y = (t^2 - t + 2)(2t^2 + 3) \Rightarrow y' = (t^2 - t + 2) \cdot 4t + (2t^2 + 3)(2t - 1) \quad \text{ب.}$$

$$\lim_{x \rightarrow -9} \frac{x^2 - 81}{x + 9} = \lim_{x \rightarrow -9} \frac{(x-9)(x+9)}{x+9} = \lim_{x \rightarrow -9} (x-9) = -9 - 9 = -18, \quad f(-9) = -18 \quad \text{أ.4}$$

أي أن $f(-9) = \lim_{x \rightarrow -9} f(x)$ ولذلك فإن الدالة متصلة

$$f'(x) = \frac{x^3 \cdot 7(4x^2 + 5)^6 \cdot 8x - (4x^2 + 5)^7 \cdot 3x^2}{x^6} = \frac{(4x^2 + 5)^6 (44x^2 - 15)}{x^4} \quad \text{ب.}$$

أ.5

$$\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2} = \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 4)}{x - 2} = \lim_{x \rightarrow 2} (x^2 + 2x + 4) = 12 = f(2) = k$$

$$\therefore k = 12$$

$$f'(t) = 120t \Rightarrow f'(5) = 120 \times 5 = 600 \quad \text{ب.}$$

$$y' = 2x^2(-5e^{-5x}) + e^{-5x} 4x = (4x - 10x^2)e^{-5x} \quad \text{أ.6}$$

$$x^2 \cdot 2y \frac{dy}{dx} + y^2 \cdot 2x - 3(x \cdot \frac{dy}{dx} + y \cdot 1) = 5 \quad \text{ب.}$$

$$2x^2 y \frac{dy}{dx} + 2xy^2 - 3x \frac{dy}{dx} - 3y = 5 \Rightarrow \frac{dy}{dx} (2x^2 y - 3x) = 3y - 2xy^2 + 5$$

$$\Rightarrow \frac{dy}{dx} = \frac{3y - 2xy^2 + 5}{2x^2 y - 3x}$$