Fall 2009 92.131 Exam 1 Solutions

1) $f'(x) = -\frac{3}{2}x^{-5/2} + e^x + \frac{1}{2}x^{-1/2}$ and so $f''(x) = \frac{15}{4}x^{-7/2} + e^x - \frac{1}{4}x^{-3/2}$

2) $g'(t) = 3t^2e^t + t^3e^t = t^2e^t(3+t)$. Horizontal tangents are where $g'(t) = 0$, so $t = -3, 0$.

3) $f'(x) = \frac{2\cos x}{e^x}$.

4) $s'(t) = \sec t(\tan^2 t + \sec^2 t)$ or $\sec t(2\sec^2 t - 1)$.

5) $$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\frac{\left[3(x+h)^2 - \frac{1}{x+h}\right] - \left[3x^2 - \frac{1}{x}\right]}{h}$$

$$= \lim_{h\to 0}\frac{3x^2 + 6xh + 3h^2 - 3x^2 + \frac{1}{x} - \frac{1}{x+h}}{h} = \lim_{h\to 0}\frac{6xh + 3h^2}{h} + \lim_{h\to 0}\frac{\frac{1}{x} - \frac{1}{x+h}}{h}$$

$$= \lim_{h\to 0} 6x + \lim_{h\to 0}\frac{\frac{x+h-x}{x(x+h)}}{h} = 6x + \lim_{h\to 0}\frac{\frac{h}{x(x+h)}}{h} = 6x + \lim_{h\to 0}\frac{1}{x(x+h)} = 6x + \frac{1}{x^2} = 6x + x^{-2}$$

Power rule is $\frac{d}{dx}x^k = kx^{k-1}$. $\frac{d}{dx}(3x^2 - x^{-1}) = \frac{d}{dx}(3x^2) - \frac{d}{dx}(x^{-1}) = 6x + x^{-2}$

6) $g'(t) = \frac{8t^3}{(t^4+3)^2}$

7) $f'(x) = (x+1)^2\cos x - (x^2 - 2x + 4)\sin x$

8) Since $f(1) = 4$, the point on the T.L. is $(1, 4)$. Since $f'(x) = 3x^2 + 2 + 4x^3$ the slope of the T.L. is $f'(1) = 9$. So $y - 4 = 9(x-1)$ and so $y = 9x - 5$

9) a) $0.054x + .076(1000) = .064(1000 + x)$ or $0.054x + 76 = 64 + 0.064x$

b) Equation can be reduced to $0.01x = 12$, or $x = 1200$cc

10 Graph the function $f(x) = 2x - x^2 + 3$ on the interval [–3, 4] below indicating scales, and labeling the vertex and all intercepts as well as end-points.

Answer: Endpoints are in green (–3, –12), and (4, –5). *x*- intercepts are (–1, 0), and (3, 0). *y*-intercept is (0, 3) in red, and vertex in blue is (1, 4).