Fall 2009 92.131 Exam 2 Solutions

1) **a)** $f'(x) = \sec^2 x e^{\tan x}$ **b)** $g'(x) = \tan x$ **c)** $s'(t) = -\frac{t\sin(\sqrt{t^2+1})}{\sqrt{t^2+1}}$

2) Consider the function $f(x) = x^4 - 18x^2 + 2$.

a) Find the intervals of increase or and decrease.

$f'(x) = 4x^3 - 36x = 4x(x^2-9) = 4x(x-3)(x+3)$ Function increases on $(-3,0)\cup(3,\infty)$. Function decreases on $(-\infty,-3)\cup(0,3)$

b) Find the local maximum and minimum values. Local mins of –79 at $x = \pm 3$. Local max of 2 at $x = 0$.

c) Find the intervals of concavity and any points of inflection.

$f''(x) = 12x^2 - 36 = 12(x^2-3) = 12(x-\sqrt{3})(x+\sqrt{3})$. Concave up on $(-\infty,-\sqrt{3})\cup(\sqrt{3},\infty)$, concave down on $(-\sqrt{3},\sqrt{3})$. Inflection points at $(\pm\sqrt{3},-43)$

3) Find the equation of the line tangent to the graph of $2xy = \pi\sin(y)$ at the point $(0, 2\pi)$. Write your answer using slope-intercept form.

Using Implicit Diff: $D_x 2xy = D_x[\pi\sin(y)]$ or $2y + 2x\frac{dy}{dx} = \pi\cos(y)\left(\frac{dy}{dx}\right)$, which at $(0,2\pi)$ becomes

$4\pi = \pi\frac{dy}{dx}$, and so $m = \frac{dy}{dx} = 4$.

Tangent Line is $y - 2\pi = 4(x-0)$, or $y = 4x + 2\pi$.

4) Compute the second derivative of $f(x) = e^{2x}\cosh(3x)$. **(**

$f'(x) = 2e^{2x}\cosh(3x) + 3e^{2x}\sinh(3x)$

$$f''(x) = 4e^{2x}\cosh(3x) + 6e^{2x}\sinh(3x) + 6e^{2x}\sinh(3x) + 9e^{2x}\cosh(3x)$$
$$= e^{2x}[12\sinh(3x) + 13\cosh(3x)]$$

5) **a)** Find the absolute minimum and maximum values of the function $f(x) = 4x^3 + 3x^2 - 6x$ on the interval [–2, 1].

Since $f'(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1) = 6(2x-1)(x+1)$ critical points are $x = \frac{1}{2}, -1$. $f(-2) = -8$, $f(-1) = 5$ $f(1/2) = -\frac{7}{4}$, and $f(1) = 1$, hence the absolute max is 5, and the absolute min is –8. FYI:

b) Compute the derivative of the function $g(t) = (t^2+2t)^{\sin t}$.

$$\ln[g(t)] = \ln[(t^2+2t)^{\sin t}] = \sin t \ln(t^2+2t)$$
$$\frac{g'(t)}{g(t)} = \cos t \ln(t^2+2t) + \frac{(2t+2)\sin t}{t^2+2t}$$
$$g'(t) = (t^2+2t)^{\sin t}\left(\cos t \ln(t^2+2t) + \frac{(2t+2)\sin t}{t^2+2t}\right)$$